

Weekly Review 2

① Find the limits algebraically then verify with your calculator.

$$\textcircled{a} \lim_{x \rightarrow 2} f(x) \quad \text{if } f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ \pi, & x = 2 \end{cases}$$

$$\textcircled{b} \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x}$$

$$\textcircled{c} \lim_{x \rightarrow \frac{\pi}{2}} \tan x \cos x$$

② Find the interval(s) over which the following function is continuous. Fully justify your answer.

$$h(x) = \begin{cases} 4x + 4, & x \leq -2 \\ 2 - x^2, & -2 < x < 0 \\ 2 \cos x, & x \geq 0 \end{cases}$$

- ③ A population of 50 bacteria is introduced into a culture and grows in number according to the equation

$$P(t) = \frac{25}{3}t^3 + 50t + 50$$

- (a) Find the average rate of change in the population from $t=0$ hours to $t=3$ hours.
- (b) Find the instantaneous rate of change in the population at $t=2$ hours.

④ Suppose $f(2) = -1$, $g(2) = 1$, $f'(2) = 4$, $g'(2) = -3$

Find the derivative at 2 of each of the following functions.

(a) $s(x) = f(x) + g(x)$

(b) $p(x) = f(x)g(x)$

(c) $q(x) = \frac{f(x)}{g(x)}$

(5) Answer the following questions by analyzing $f(x)$ algebraically then verify your answers with your calculator.

$$f(x) = -16x^2 + 160x - 256$$

(a) Does the graph open upward or downward?

(b) What is the y-intercept?

(c) What is the x-int?

(d) What is the range of the function?

(e) Find the vertex.

(f) At what x-values does $f(x) = 80$?

(g) For what x-value(s) does $\frac{d}{dx}f(x) = 100$?

(h) Find $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$

(i) on what interval is $\frac{d}{dx}f(x) > 0$?

(j) Find $\frac{d^2}{dx^2}f(x)$ at $x=7$

Answers

① a 4

b $\frac{3}{5}$

c 1

② discontinuous at $x = -2$, b/c one-sided limits are different

③ a 125

b 150

④ a 1

b 7

c 1

⑤

a Downward b -256 c 2, 8 d $(-\infty, 144]$ e $(5, 144)$

f $x = 3, 7$

g $x = \frac{15}{8}$

h $(-\infty, 5)$

i 64

j -32